# Optimal view angle in collective dynamics of self-propelled agents

Bao-Mei Tian,[1] Han-Xin Yang,[1] Wei Li,[2] Wen-Xu Wang,[3] Bing-Hong Wang,[1,4] and Tao Zhou[1,5,*]

[1]*Department of Modern Physics, University of Science and Technology of China, Hefei 230026, China*

[2]*Department of Electrical Engineering, University of Texas at Dallas, Richardson, Texas 75083, USA*

[3]*Department of Electronic Engineering, Arizona State University, Tempe, Arizona 85287-5706, USA*

[4]*The Research Center for Complex System Science, University of Shanghai for Science and Technology, Shanghai 200093, China*

[5]*Department of Physics, University of Fribourg, Chemin du Musée 3, CH-1700 Fribourg, Switzerland*

(Received 22 June 2008; revised manuscript received 4 January 2009; published 18 May 2009; corrected 29 May 2009)

We study a system of self-propelled agents with the restricted vision. The field of vision of each agent is only a sector of disk bounded by two radii and the included arc. The inclination of these two radii is characterized by the view angle. The consideration of restricted vision is closer to the reality because natural swarms usually do not have a panoramic view. Interestingly, we find that there exists an optimal view angle, leading to the fastest direction consensus. The value of the optimal view angle depends on the density, the interaction radius, the absolute velocity of swarms, and the strength of noise. Our findings may invoke further efforts and attentions to explore the underlying mechanism of the collective motion.

DOI: 10.1103/PhysRevE.79.052102 PACS number(s): 05.60.Cd, 87.10.−e, 89.75.Hc, 02.50.Le

The collective motion of a group of autonomous agents (or particles) [1–8] has attracted much attention in the past decade. One of the most remarkable characteristics of systems, such as flocks of birds, schools of fish, and swarms of locusts, is the emergence of collective states in which the agents move in the same direction. A particularly simple and popular model to describe such behavior was proposed by Vicsek *et al.* [9]. Due to simplicity and efficiency, the Vicsek model (VM) has been intensively investigated in recent years [10–22].

In the VM, $N$ agents move synchronously in a square-shaped cell of linear size $L$ with the periodic boundary conditions. The initial directions and positions of the agents are randomly distributed in the cell, and each agent has the same absolute velocity $v_0$. Agents $i$ and $j$ are neighbors at time step $k$ if and only if $\|\vec{X}_i(k)-\vec{X}_j(k)\|\leq R$, where $\vec{X}_i(k)$ denotes the position of agent $i$ on a two-dimensional (2D) plane at time step $k$ and $R$ is the sensor radius. The direction of agent $i$ at time step $k+1$ is

$$\theta_i(k+1)=\langle\theta_i(k)\rangle_R+\Delta\theta, \tag{1}$$

where $\langle\theta_i(k)\rangle_R$ denotes the average direction of agent $i$'s neighbors (include itself), $\Delta\theta$ denotes noise (in the following discussions, $\Delta\theta=0$ without special mention). To be more specific, let $\Gamma_i(k)$ be the set of neighbors of agent $i$ at time step $k$, the VM is then described as [16,17]

$$\vec{X}_i(k+1)=\vec{X}_i(k)+v_0 e^{i\theta_i(k)}\Delta t, \tag{2}$$

$$\theta_i(k+1)=\text{angle}\left(\sum_{j\in\Gamma_i(k+1)} e^{i\theta_j(k)}\right), \tag{3}$$

where $e^{i\theta_i(k)}$ is the unitary complex directional vector of agent $i$, $e^{i\theta_i(k)}=\cos(\theta_i(k))+i\sin(\theta_i(k))$, where $\theta_i(k)\in[0,2\pi)$. Here the function angle$(\cdot)$ denotes the angle of a complex number. $\theta_i(k+1)$ is the moving direction of agent at time step $k+1$, which is the average direction of agents in the neighbor set $\Gamma_i(k+1)$. $v_0 e^{i\theta_i(k)}$ represents the velocity of agent $i$ at time step $k$ with constant speed $v_0$ and direction $\theta_i(k)$.

In the VM and most other models of self-propelled particles, the field of vision for every agent is a complete disk (2D case) or a sphere [three-dimensional (3D) case] characterized only by its sensor radius $R$. In reality, however, most animals are incapable of complete view. For example, the cyclopean retinal field of human is about 180° and the cyclopean retinal field of tawny owl is 201° [23]. It is thus more reasonable to assume limited view angles of agents [3,24], instead of the omnidirectional views, in swarm models to better mimic the real collective behaviors.

In this Brief Report, we investigate the VM in which agents have limited view angles $\omega$, with $\omega\in(0,2\pi]$. As illustrated in Fig. 1, the field of vision of every agent is only a sector of disk bounded by two radii and the included arc, the left (right) boundary of vision and the heading of agent $i$ have inclination $\omega/2$, that is, for every agent, the field of

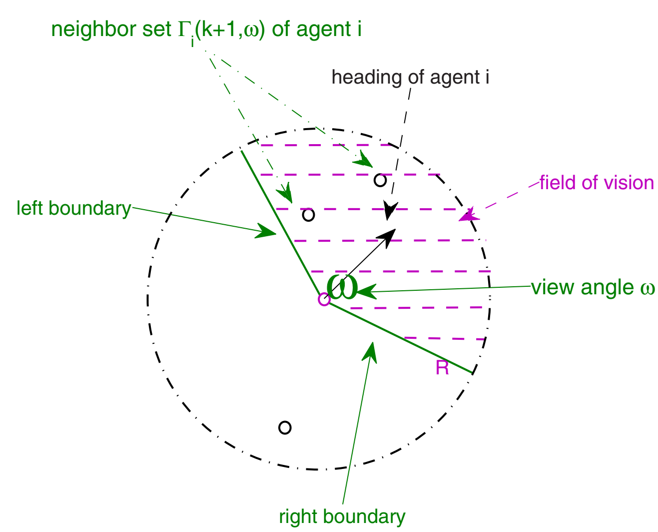


FIG. 1. (Color online) Illustration of the nonomnidirectional view of agent $i$ at time step $k+1$ in a 2D plane.

*Corresponding author: zhutou@ustc.edu.

 

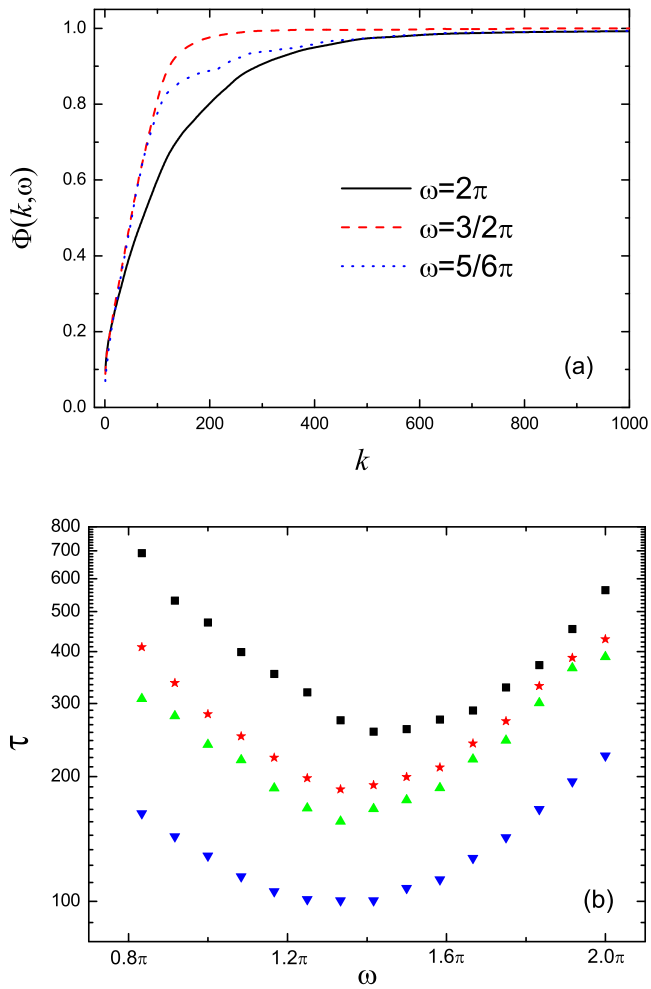


FIG. 2. (Color online) (a) The order parameter $\Phi(k,\omega)$ as a function of time step $k$ for different values of view angle $\omega$. Here $N$=400, $R$=0.6, and $v_0$=0.04. (b) The transient time step $\tau$ as a function of the view angle $\omega$. The symbols correspond to ■: $R$=0.6, $v_0$=0.02, $N$=400; ★: $R$=0.6, $v_0$=0.04, $N$=400; ▲: $R$=0.6, $v_0$=0.04, $N$=500; ▼: $R$=0.8, $v_0$=0.04, $N$=400. Each data point is obtained by averaging over 500 different realizations.

view is symmetric about its current moving direction. Thus rule (3) in the VM can be modified as

$$\theta_i(k+1) = \text{angle}\left(\sum_{j\in\Gamma_i(k+1,\omega)} e^{i\theta_j(k)}\right), \tag{4}$$

where $\Gamma_i(k+1,\omega)$ denotes the neighbor set of agent $i$ with view angle $\omega$. When $\omega=2\pi$, rule (4) degenerates to the original Vicsek model (3).

To give a quantitative discussion, we define an order parameter

$$\Phi(k,\omega) = \frac{1}{N}\left|\sum_{i=1}^{N} e^{i\theta_i(k)}\right|, \quad 0 \le \Phi(k,\omega) \le 1, \tag{5}$$

for system (4) at time step $k$ with view angle $\omega$, obviously, $0\le\Phi(k,\omega)\le 1$.

In noiseless case, the order parameter $\Phi(k,\omega)$ can approach 1 when the evolution is long enough, except for extremely rare cases (for example, the cases may occur when $R$ or $\omega$ is too small). To quantify the speed of direction consensus, we study the transient time step $\tau$, which is defined as the time step when the order parameter first surpasses a certain value $\Phi_0$. Here we take $\Phi_0$=0.99 and we have checked that qualitative results are not changed when $\Phi_0$ is large enough.

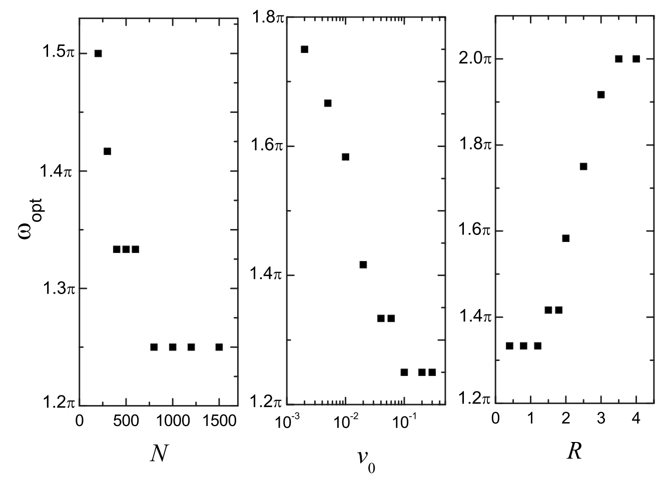


FIG. 3. The optimal view angle $\omega_{opt}$ as functions of the swarm number $N$, sensor radius $R$, and absolute velocity $v_0$, respectively. For the left panel: $R$=0.6, $v_0$=0.04; for the middle panel: $R$=0.6, $N$=400; and for right panel: $v_0$=0.04, $N$=400. The lattice size is fixed as $L$=10. Each data point is obtained by averaging over 500 different realizations. Note that the resolution of view angle in our simulation is set to be $\pi/12$.

We then investigate the effects of the view angle $\omega$ on the transient process. As shown in Fig. 2(a), the order parameter $\Phi(k,\omega)$ reaches 1 faster when the view angle $\omega=3\pi/2$, compared with $\omega=2\pi$ and $\omega=5\pi/6$. Figure 2(b) shows the transient time step $\tau$ as a function of $\omega$ for different values of parameters. One can find that $\tau$ is not a monotonic function of $\omega$ and there exists an optimal view angle, leading to the shortest transient time.

Figure 3 shows the optimal view angle $\omega_{opt}$ as functions of the parameters: the swarm number $N$, the sensor radius $R$, and the absolute velocity $v_0$, respectively. One can see that the optimal view angle $\omega_{opt}$ decreases with the increasing of $N$ and $v_0$, and converges to a fixed value when $N$ or $v_0$ is large enough. $\omega_{opt}$ increases as the sensor radius $R$ increases. In particular, when $R$ is close to the lattice size $L$, agents with panoramic view will be globally coupled and the directions of the swarm can reach consensus in only one time step.

We next investigate whether more communications are needed for faster convergence. We define $n_i(k,\omega)$ as the number of $i$'s neighbors, and the average number of neighbors $\langle n(k,\omega)\rangle$ over all agents at time step $k$ is

$$\langle n(k,\omega)\rangle = \frac{1}{N}\sum_{i=1}^{N} n_i(k,\omega). \tag{6}$$

In Fig. 4, we report this average neighboring number for different $\omega$. Combining Figs. 2(a) and 4, it is interesting to find that agents with optimal view angle $\omega=3\pi/2$ have the

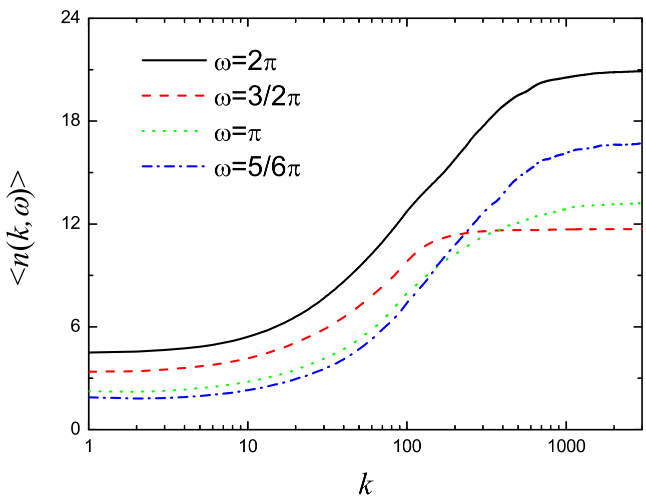


FIG. 4. (Color online) The average number of neighbors $\langle n(k,\omega)\rangle$ as a function of time step $k$ for different view angle $\omega$. Here the parameters $N$, $L$, $R$, and $v_0$ are the same with the parameters in Fig. 2(a). Each data point is obtained by averaging over 500 different realizations.

least number of neighbors in the steady state, compared with $\omega=2\pi$, $\omega=5\pi/6$, and $\omega=\pi$. This result indicates the existence of superfluous communications in the VM, which may counteract the direction consensus.

In the following, we focus on the noise effects associated with the restriction of view angle. The noise is introduced to the view angle model as

$$\theta_i(k+1) = \text{angle}\left(e^{i\xi} \sum_{j\in\Gamma_i(k+1,\omega)} e^{i\theta_j(k)}\right), \tag{7}$$

where the moving direction of each agent is perturbed by a random number $\xi$ chosen with a uniform probability from the interval $[-\eta,\eta]$. In the presence of noise, the order parameter $\Phi(k,\omega,\eta)$ will fluctuate and never remain fixed at a certain value; therefore we adopt a statistically stable order parameter in terms of $\Phi_{stable}(\omega,\eta)$, which is an average of the consecutive series of $\Phi(k,\omega,\eta)$ over many time steps after a sufficiently long transient time. Figure 5 shows that $\Phi_{stable}(\omega,\eta)$ increases as $\omega$ increases if the noise is kept constant and decreases as the noise increases.

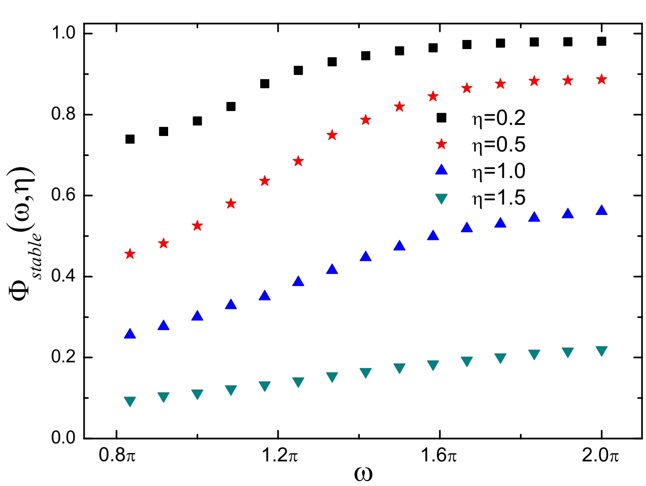


FIG. 5. (Color online) The statistically stable order parameter $\Phi_{stable}(\omega,\eta)$ as a function of the view angle $\omega$ for different noise $\eta$. Here $\Phi_{stable}(\omega,\eta)=\frac{1}{500}\Sigma_{k=2501}^{3000}\Phi(k,\omega,\eta)$. $N=400$, $L=10$, $R=0.6$, $v_0=0.04$. Each data point is obtained by averaging over 500 different realizations.

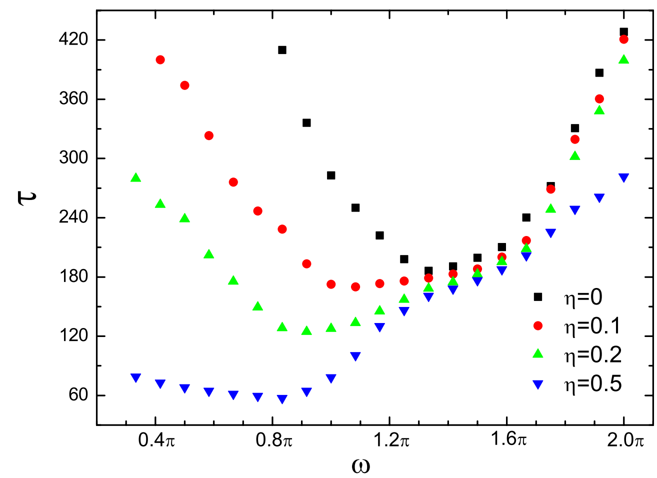


FIG. 6. (Color online) The transient time step $\tau$ as a function of the view angle $\omega$ for different values of the noise $\eta$. $N=400$, $L=10$, $R=0.6$, $v_0=0.04$. Each data point is obtained by averaging over 500 different realizations.

In the noisy case, we define the transient time step $\tau$ as the time step when the order parameter first exceeds $0.99\Phi_{stable}(\omega,\eta)$ for each run. For $\eta=0$, $\Phi_{stable}(\omega,0)$ approaches 1; thus this definition of $\tau$ is applicable in the absence of noise. From Fig. 6, one can find that there still exists an optimal view angle $\omega_{opt}$ leading to the shortest transient time step in the presence of noise and the value of the optimal view angle decreases as the noise increases.

In conclusion, we have studied the effects of restricted vision of a group of self-propelled agents. The field of vision of every agent is only a sector of disk and the included arc represents the view angle. It is interesting to find that there exists an optimal angle resulting in the fastest direction consensus. The value of the optimal view angle increases as the sensor radius increases, while it decreases as the swarm number, the absolute velocity, or the noise strength increases. Another interesting phenomenon is that agents with optimal view angle have the least number of neighbors in the steady state. Our studies indicate the existence of superfluous communications in the Vicsek model, which indeed hinder the direction consensus. Moreover, our results may be useful in designing the manmade swarms such as autonomous mobile robots.

We thank Hai-Tao Zhang and Ming Zhao for their valuable comments. This work was funded by the National Basic Research Program of China (973 Program No. 2006CB705500), the National Natural Science Foundation of China under Grants No. 10635040 and No. 10805045, and the Specialized Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20060358065).

[1] J. K. Parrish, Science **284**, 99 (1999).
[2] H. Levine, W. J. Rappel, and I. Cohen, Phys. Rev. E **63**, 017101 (2000).
[3] I. D. Couzin, J. Krause, R. James, G. D. Ruxton, and N. R. Franks, J. Theor. Biol. **218**, 1 (2002).
[4] I. D. Couzin, J. Krause, N. R. Franks, and S. A. Levin, Nature (London) **433**, 513 (2005).
[5] J. Buhl, D. J. T. Sumpter, I. D. Couzin, J. J. Hale, E. Despland, E. R. Miller, and S. J. Simpson, Science **312**, 1402 (2006).
[6] M. R. D'Orsogna, Y. L. Chuang, A. L. Bertozzi, and L. S. Chayes, Phys. Rev. Lett. **96**, 104302 (2006).
[7] D. Grunbaum, Science **312**, 1320 (2006).
[8] A. Kolpas, J. Moehlis, and I. G. Kevrekidis, Proc. Natl. Acad. Sci. U.S.A. **104**, 5931 (2007).
[9] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, Phys. Rev. Lett. **75**, 1226 (1995).
[10] L. Moreau, IEEE Trans. Autom. Control **50**, 169 (2005).
[11] F. Cucker and S. Smale, IEEE Trans. Autom. Control **52**, 852 (2007).
[12] G. Grégoire and H. Chaté, Phys. Rev. Lett. **92**, 025702 (2004).
[13] C. Huepe and M. Aldana, Phys. Rev. Lett. **92**, 168701 (2004).
[14] M. Aldana, V. Dossetti, C. Huepe, V. M. Kenkre, and H. Larralde, Phys. Rev. Lett. **98**, 095702 (2007).
[15] M. Nagy, I. Daruka, and T. Vicsek, Physica A **373**, 445 (2007).
[16] W. Li and X. F. Wang, Phys. Rev. E **75**, 021917 (2007).
[17] W. Li, H. T. Zhang, M. Z. Q. Chen, and T. Zhou, Phys. Rev. E **77**, 021920 (2008).
[18] W. Li, IEEE Trans. Syst., Man, Cybern., Part B: Cybern. **38**, 1084 (2008).
[19] H. Chaté, F. Ginelli, G. Grégoire, and F. Raynaud, Phys. Rev. E **77**, 046113 (2008).
[20] H. T. Zhang, M. Z. Q. Chen, and T. Zhou, Phys. Rev. E **79**, 016113 (2009).
[21] L. Q. Peng, Y. Zhao, B. M. Tian, J. Zhang, B. H. Wang, H. T. Zhang, and T. Zhou, Phys. Rev. E **79**, 026113 (2009).
[22] J. Zhang, Y. Zhao, B. M. Tian, L. Q. Peng, H. T. Zhang, B. H. Wang, and T. Zhou, Physica A **388**, 1237 (2009).
[23] G. R. Martin, J. Comp. Physiol., A **174**, 787 (1994).
[24] A. Huth and C. Wissel, J. Theor. Biol. **156**, 365 (1992).